The fuzzy-AI Modeling for Optimization of Long-Term Metro Vehicle Repair

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Abstract

In this paper, the “Fuzzy-AI model” is applied for investment decision of optimum repair of multiple metro vehicles. The model solves this highly uncertain and sophisticated problem both based on its fuzzy analysis and reasoning of existing distressed metro vehicles. The VDSI (vehicle distress state indicator) is used in finding the optimum investment management of repair, which is a fuzzy synthesis of quantized indicator for the distressed vehicle. An optimization of long-term vehicle repair investment planning is suggested.

1. Introduction

Due to rapid urbanization development, rail transportation is considered as the main tool for solving the urban traffic problem. It is obvious that there is a tremendous and persistent work for vehicle maintenance and repair in order to maintain a metro network system in normal operation.

According to estimation, the cost for regular repair consists of one third operational cost of the metro operation; it seems worthwhile to improve the repair strategy of the metro vehicle and the problems concerning to optimization of repair arrangement are:

(1) How to quantitatively evaluate the damage level of an existing distressed metro vehicle by certain synthetic indicator?
(2) How to invest annual funds for maintenance and repair of metro vehicle so as to establish optimum long-term reliable system of vehicle serviceability?

Eventually, VDSI is a quantized indicator for appraise the distress condition by detecting measurement. As the uncertain feature of the measuring data, VDSI is an uncertain value which is appropriately synthesized by in-deterministic methods. Therefore, the “Fuzzy-AI model” is used in this paper for manipulating the distresses of different attributes of the vehicle caused by extremely complicated working conditions of vehicle operation, such as mechanical faults, wear damage, cracking, fatigues and human faults, overloading, wrong operation etc.

2. Theoretical Analyses

2.1. The State Indicator VDSI of Vehicle

A fuzzy decision of repair planning is based on the Fuzzy-AI Model; which is quantized by synthesis of each attributive fuzzy distance of distressed vehicle. Assuming S is the state indicator of VDSI, representing the damage degree of the vehicle:

\[ S=F(t_1,t_2,\ldots,t_m)=F(t_i) \]  \hspace{1cm} (1)

Where \( t_i \)\((i=1, 2,\ldots, m)\) is attributes, for distressed vehicle, \( t_i \) represent different faults in cracking, wearing degree, fatigue of components respectively. Suppose the vehicle is out of work if any \( t_i \) approaches to its critical value \( t_i^{cr} \), then the failure state expressed by fuzzy theory can be
defined by fuzzy membership function \( \mu^{(i)} = 1 \) as shown in Fig.1. It is noticed that the membership function in Fig.1 is just empirical; its improvement can be realized by machine learning shown in [3].

\[
\mu(t_i) = \begin{cases} 
1 & \text{if } t_i < d_i < d_i^c \\
0 & \text{if } d_i \geq d_i^c \text{ or } d_i \leq 0
\end{cases}
\]

Figure 1. Membership Function

Introducing the concept of fuzzy distance between state \( \tilde{A} \) and state \( \tilde{B} \) for expressing state characteristics by fuzzy approach:

\[
D(\tilde{A}, \tilde{B}) = \sum_{i=1}^{m} w_i \cdot d_i(\tilde{A}, \tilde{B})
\]

where \( m \) — the dimension of attribute space of distressed vehicle states \( \tilde{A} \) and \( \tilde{B} \);

\( w_i \) — the weight of the i-th attribute, then

\[
d_i(\tilde{A}, \tilde{B}) = \begin{cases} 
|\mu_{\tilde{A}}(t_i) - \mu_{\tilde{B}}(t_i)| & \text{if } \mu_{\tilde{A}}(t_i) < \mu_{\tilde{B}}(t_i) \\
0 & \text{if } \mu_{\tilde{A}}(t_i) \geq \mu_{\tilde{B}}(t_i)
\end{cases}
\]

where \( \mu_{\tilde{A}}(t_i) \) and \( \mu_{\tilde{B}}(t_i) \) represent the value of fuzzy membership function of i-th attribute in state \( \tilde{A} \) and \( \tilde{B} \); \( d_i^c \) and \( d_i \) represent the upper bound and lower bound of \( d_i \). The i-th attribute and its varied weight function of weight factor \( a_i \) is

\[
w_i(d_i) = H_i(a_i, d_i) \cdot \zeta_i(d_i)
\]

\[
\zeta_i(d_i) = \left(1 - 2d_i + d_i^2\right)
\]

\[
H_i(a_i, d_i) = \left((1 - a_i) \exp\left(-d_i^2 / p\right) + a_i\right)
\]

\( H_i \) -- the amplifier function of critical state \( \zeta_i \) -- the penalty of fuzzy distance as shown in Fig.2.

\[
P \text{-- amplifying factor; } p = 0.1 \text{ is defined empirically; the summation of } a_i (i=1, 2, \cdots, m) \text{ is unity.}
\]

It is evident from Fig.2, if \( d_i \) vanishes, \( \zeta_i(d_i) \) and \( H_i(a_i, d_i) \) approach to 1, then \( w_i(d_i) \) tends to 1.0. However, when \( d_i \) approaches 1.0, then \( \zeta_i(d_i) \) will tend to zero. In Fig.2 the curve \( H_i \) represents the variation of weight factor \( a_i \) of the i-th attribute \( t_i \), when \( d_i \) approaches zero, \( H_i \) tends to 1.0. The penalty function \( \zeta_i \) contains two extreme conditions; when \( \zeta_i \) tends to 1.0, \( d_i \) vanishes; and when \( \zeta_i \) vanishes, \( t_i \) is in normal state.

2.2. The Repair Investment for Distressed Vehicle

The VDSI of vehicle can be obtained as long as the membership functions of each distress attribute are known. The fuzzy distance of existing vehicle with the perfect one, can be determined by (5); for the VDSI of vehicle is the weighted average sum of the individual attributive distance
\(d_i(A,B)\) of the vehicle itself.

\[
\{\text{VDSI}\}_\text{veh} = \sum_{i=1}^{m} \{w_i(t_i) \cdot d_i(A,B)\} \quad (5)
\]

where \(\{\text{VDSI}\}_\text{veh}\)---- VDSI for individual vehicle
\(w_i(t_i) = \) weight factor of \(i\)th attribute \((i=1,2,\ldots,m)\)

Define \(S\) the real time VDSI of the vehicle, which represents the state of serviceability of the vehicle. Fig.3 is the state chronicle chart of \(S\) across time. If repair investment is putting in for an existing distressed vehicle, there must be a deduction of \(S\) as shown by \(\Delta S\) in Figure 3. There are \(\Delta S_1 > \Delta S_2 > \Delta S_3\), representing the VDSI improvements for capital repair, moderate repair and maintenance repair, respectively.

Obviously, fuzzy distance in (5) is the synthesis of fuzzy sub-distance of different attributes. As pointed by (1), for quantizing \(S\), one should provide qualified membership function for each attributes \(t_i\). Practically, we could just adopt some most important attributes for \(S\) and the \(d_i\) satisfies the first condition of (3). Therefore, we have:

Where \(\tilde{S}\) and \(\bar{S}\) represents the upper and lower bounds of \(S\). Actually, \(S\) is a comprehensive state indicator which could control the system (the vehicle) performance properly. In Fig.3, the \(S\) chronicle chart of the vehicle, \(S > \tilde{S}\) means the vehicle is exceedingly defective, and all operation should be halted; \(\bar{S} > S > \tilde{S}\) means the vehicle is in good condition and there is no need to repair; \(\bar{S} > S > \tilde{S}\) represents a state during which the repair should be processed. \(\Delta S_1, \Delta S_2,\) and \(\Delta S_3\) represent the improvements of \(S\) by decreasing the generalized state \(S\) of the vehicle in \(\Delta S_1, \Delta S_2,\) and \(\Delta S_3\) due to different intensities of repair investment.

3. Investment Decision Making for vehicles

Once the VDSI upper and lower bounds \((\tilde{S} \text{ and } \bar{S})\) are defined, sequentially the fuzzy decision model for repair planning can be defined in Figure 3. Under definite repair investment and definite duration of time, the repair planning should maintain the service duration of each vehicle as long as possible in the repair zone \((\bar{S} > S > \tilde{S})\). It is a typical problem of decision making for system optimization. The optimization problem can be stated so as to minimize the weighted average sum of VDSI in (5) with the constraints of definite repair investment under definite time duration.

3.1 VDSI Calculation

In reality, the repair of vehicle for metro network is under multiple vehicles condition, the \(\{\text{VDSI}\}_\text{group}\) of multiple vehicles (comprised of \(r\) vehicles) can be determined by (9) as the weighted average sum of VDSI of individual vehicles;

For \(p\)-th vehicle, the VDSI after repair is

\[
\{\text{VDSI}\}_p = \tilde{S}_j - \Delta S_j = \{ \sum_{i=1}^{m} \{S_{ij} - \Delta S_{ij}\} \}_p = \{ \sum_{i=1}^{m} \{w_{ij}d_{ij} - w_{ij}\Delta d_{ij}\} \}_p \quad (9)
\]

where \(\Delta S_{ij}\), the improvement of \(S\) in \(i\)-th attribute of \(p\)-th vehicle after repair; By (3) the fuzzy distance after repair is:
where \( d_{ij} \), the fuzzy distance of i-th attribute for j-th vehicle
\[ u(t^c) - u(t_{ij}) \]

(10)

\[ \text{where } d_{ij}, \text{ the fuzzy distance of i-th attribute for j-th vehicle} \]
\[ u(t^c) \text{ the membership function value of attribute } t \text{ when approaches to critical value, we have } u(t^c) = 1.0 \]
\[ u(t_{ij}) \text{ the membership function value of i-th attribute for j-th vehicle} \]

The fuzzy distance change of i-th attribute for j-th vehicle after repair
\[ \Delta d_{ij} = u(t^c) - u(t_{ij} - \Delta t_{ij}) \]

(11)

\[ \text{The change of i-th attribute for j-th vehicle after repair} \]
\[ \Delta t_{ij} = t_{ij} - \Delta t_{(g)ij} \]

(12)

\[ \text{where } \Delta t_{(g)ij}, \text{ the decrease of } t_{ij} \text{ by g-th grade of repair} \]
\[ g = 1, 2, 3; \text{ with 1 – Capital repair; 2 – Regular repair and 3 – maintenance.} \]

Substituting (10), (11) and (12) to (9), we have
\[ (VDSI)_j = \frac{1}{m_i} \sum_{i=1}^{m} \{ w_i [ d_{ij} - (u(t^c) - u(t_{ij} - \Delta t_{ij})) ] \} \]

(13)

\[ \text{where } u_i \text{ is the fuzzy membership function of i-th attribute.} \]

For a metro system A with k vehicles
\[ (VDSI)_A = \sum_{j=1}^{k} \frac{1}{m_i} \sum_{i=1}^{m} \{ w_i [ d_{ij} - (u(t^c) - u(t_{ij} - \Delta t_{ij})) ] \} \]

(14)

For a metro system B with q kinds of vehicles, each kind possesses \( k_j \) vehicles
\[ (VDSI)_B = \sum_{p=1}^{q} (VDSI)_{A_p} \]
\[ = \sum_{j=1}^{k} \sum_{i=1}^{m} \{ w_i [ d_{ij} - (u(t^c) - u(t_{ij} - \Delta t_{ij})) ] \} \]

(15)

\[ \text{Where (15) means the total VDSI of metro system B is } (VDSI)_B, \text{ which is the summation of VDSI of each vehicle group } A_1, A_2, \ldots A_q \text{ with different kinds of vehicle and numbered by } k_1, k_2, \ldots, k_q \text{ respectively.} \]

### 3.2 Repair Investment Analysis

Essentially, the influence of repair to VDSI of vehicle is reflected in the second term of (13), where the deduction of attributive value \( \Delta t_{(g)ij} \) by repair is an integer (when there is only one kind of vehicle, it can be expressed by \( \Delta t_{(g)ij} \)). When the repair grade \( g \) is defined, we have
\[ \Delta t_{(g)ij} = (\text{Const})_{ig} \]

(16)

The (16) means the deduction of VDSI for i-th attribute after repair is \( \Delta t_{(g)ij} \), which is a constant \( (\text{Const})_{ig} \) depended on the selection of repair grade \( g \), where \( g \) could be chosen optionally by capital repair (take 1), regular repair (take 2) or maintenance (take 3) respectively. The \( 3m \) constants \( (\text{Const})_{ig} \) \( (i=1,\ldots,m; g=1,2,3) \) could be accordingly defined in advance. It is also mentioned that, the selection of \( g \) is performed automatically by the model, and the \( (\text{Const})_{ig} \) are different from each attribute \( i \).

### 3.3 Optimization modeling for vehicle system with same kind of vehicles

Recall (14) the optimization of repair investment is formulated for metro system \( A \) with \( k \) vehicles, we could formulate the optimization modeling by mathematical programming
\[ \text{Min. } (VDSI)_A = \sum_{j=1}^{k} \sum_{i=1}^{m} \{ w_i [ (1 - u_{ij}(t_{ij})) - (1 - u_{ij}(t_{ij} - \Delta t_{(g)ij})) ] \} \]

(17-1)

Find \( 3mk \) integer \( G_{ijg} \) \((i=1,2,\ldots, m; j=1,2,..,k; \text{for } g=1,2 \text{ or } 3) \)

Subject to: Cost constraints
\[ \sum_{j=1}^{k} C_j < = C_o \]

(17-2)

Manpower constraint
\[ \sum_{j=1}^{k} [\text{MP}]_j < = [\text{MP}]_o \]

(17-3)

In (17), \( C_o \) and \( C_j \) represent annual cost budget limit and cost for j-th vehicle repair; \( [\text{MP}]_o \) and \( [\text{MP}]_j \) represent annual manpower limit and manpower for j-th vehicle repair respectively.

The formulation of (17) is not yet a
standardized form of optimization. In order to establish the relation of $\Delta t_{ig}$ in (16) with (17-2) for cost estimation, we have the cost estimation for $i$-th attribute under different g-grade of repair

$$(C_{ig}) = (\text{Constant})_{ig} \quad (i=1,2,\ldots,m; \; g=1,2,3)$$

(18)

Here $(\text{Constant})_{ig}$ are 3m constants which are defined in advance. The (17-2) becomes

$$\sum_{j=1}^{k} \left[ \sum_{i=1}^{m} C_{ig} \right]_{j} = C_{o} \quad (i=1,2,\ldots,m; \; j=1,2,\ldots,k; \; g=1,2,3)$$

(19)

Here $C_{ig}$ is the cost necessary for $i$-th attribute in taking g-grade of repair.

$$\sum_{i=1}^{m} C_{ig}$$

is the repair cost of $j$-th vehicle; so (19) represents the total cost of repair for $k$ vehicles will not exceed $C_{o}$.

In order to establish the relation of $\Delta t_{ig}$ in (16) to manpower consume for $i$-th attribute in g-grade repair, similar to (19), we have manpower consume $[\text{MP}]_{ig}$ for $i$-th attribute in different g-grade of repair

$$[\text{MP}]_{ig} = (\text{Constant})_{ig} \quad (i=1,2,\ldots,m; \; g=1,2,3)$$

(20)

Here $(\text{Constant})_{ig}$ are another 3m constants for manpower consume, (17-3) becomes

$$\sum_{j=1}^{k} \left[ \sum_{i=1}^{m} (\text{MP})_{ig} \right]_{j} \leq (\text{MP})_{o} \quad (i=1,2,\ldots,m; \; j=1,2,\ldots,k; \; g=1,2,3)$$

(21)

Here $(\text{MP})_{ig}$ is the manpower consumed for $i$-th attribute in g-grade repair.

4. Conclusion Remarks

It is due to the uncertain characteristics of the vehicle repair problem, the decision of its investment needs highly human intelligence. It is also why the “Fuzzy-Al Model” can be exactly applied for solving this decision making under uncertainty problem.

The optimization of investment decision is solved by non-linear integer programming set forth by (17-1), (19) and (21) for metro system with $k$ vehicles of same kind; and by (22), (23) and (24) for metro system with $q$ kinds of vehicles, each kind of vehicle group $A_{1}, A_{2}, \ldots A_{q}$ possesses $k_{1}, k_{2},\ldots k_{q}$ vehicles respectively.

There are still some details in the problem implementation; the numerical approach is suggested to solve the problem rather than proceeding of an analytical approach. The (3mk) planning variable integer $G_{ijg} \quad (i = 1,2,\ldots, m; \; j=1,2,\ldots,k; \; for \; g = 1,2 \text{ or } 3) \text{ (same kind of vehicle)}$ and (3mkq) planning variable integer $G_{ijgp} \quad (i = 1,2,\ldots, m; \; j=1,2,\ldots,k; \; p=1,2,\ldots,q \; for \; g = 1,2 \text{ or } 3) \text{ (q kind of vehicle)}$ form 3-dimensional and 4-dimensional variable vectors, which are solution vectors.

References


